

The $R_h = ct$ universe and quintessence

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ABSTRACT

Over the last few years the $R_h = ct$ universe has received a lot of attention, particularly when observational evidence seems to favor this over the standard Λ cold dark matter (Λ CDM) universe. Like the Λ CDM, the $R_h = ct$ universe is based on a Friedmann-Robertson-Walker (FRW) cosmology where the total energy density ρ and pressure p of the cosmic fluid contain a dark energy component besides the usual (dark and baryonic) matter and radiation components. However unlike the Λ CDM this model has the simple equation of state (EOS) $\rho + 3p = 0$, i.e., its total active gravitational mass vanishes, which would therefore exclude a cosmological constant as the source of its dark energy component. Faced with this issue, in this paper we examine various possible sources for the dark energy component of the $R_h = ct$ universe and show that quintessence which has been used in other various dynamical dark energy models could also be a possible source in this case.

Key words: gravitation – cosmological parameters – dark energy – cosmology: theory

1 Introduction

One of the first and simplest proposed Friedmann-Robertson-Walker (FRW) cosmological model is the Λ cold dark matter (Λ CDM) universe, which involves Einstein’s cosmological constant Λ . This standard model of cosmology, which is also referred to as the concordance model, assumes that the total energy density ρ of the universe is made up of three components, namely matter ρ_m (baryonic and dark matter), radiation ρ_r , and dark energy or vacuum energy ρ_Λ , which produces the necessary gravitational repulsion. In this model, dark energy which has an equation of state (EOS) $\omega_\Lambda = p_\Lambda/\rho_\Lambda = -1$, is a property of the space itself and its density $\rho_\Lambda = -p_\Lambda = \Lambda c^4/8\pi G$ is constant, such that as the universe expands the constant vacuum energy density will eventually exceed the matter density of the universe which is ever decreasing. The spatially flat Λ CDM model dominated by vacuum energy with $\Omega_\Lambda \sim 0.70$, with the rest of the energy density being in the form of nonrelativistic cold dark matter with $\Omega_m \sim 0.25$ and nonrelativistic baryonic matter with $\Omega_b \sim 0.05$, fits observational data reasonably well (Riess et al. 1998; Permuter et al. 1999; Knop et al. 2003; Riess et al. 2004). However the main problem in this model is the huge difference of about 10^{120} orders of magnitude between the observed value of the cosmological constant and the one predicted from quantum field theory; known as the cosmological constant problem (Weinberg 1989). Another issue is the so called coincidence problem which expresses the fact that although in this model the matter and dark

energy components scale differently with redshift during the evolution of the universe, both components today have comparable energy densities, and it is unclear why we happen to live in this narrow window of time.

Besides these main issues, there are other inherent problems faced by the Λ CDM, some of which arose as a result of recent observations that are in disagreement with the model’s predictions. For example in order to account for the general isotropy of the cosmic microwave background (CMB), the standard model invokes an early period of inflationary expansion (Kazanas 1980; Guth 1981; Linde 1982). However the latest observations by *Planck* (Ade et al. 2003) indicate that there may be some problems with such an inflationary scenario (Ijjas et al. 2013; Guth et al. 2014). It was partly due to these issues of the standard Λ CDM, that during the last decade several alternative dark energy models have been proposed and tested with observations. In these models the dark energy density component ρ_{de} is not constant and in most cases $\omega_{de} = p_{de}/\rho_{de}$ depends on time, redshift, or scale factor. For example in some of these so called dynamical dark energy models, late time inflation is achieved using a variable cosmological term $\Lambda(t)$ (Ray et al. 2011; Basilakos 2015) sometimes taken in conjunction with a time dependent gravitational constant $G(t)$ (Ray et al. 2007; Ibotombi Singh et al. 2013). Other sources of dark energy include scalar fields such as quintessence (Peebles & Ratra 2003), K-essence (Armendariz-Picon et al. 2001) and phantom fields (Singh et al. 2003). An alternative approach to the dark energy problem relies on the modification of Einstein’s theory itself such that in these alternative theories of gravity, cosmic acceleration is not provided

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solely by the matter side $T_{\mu\nu}$ of the field equations, but also by the geometry of spacetime. These theories include the scalar tensor theory with non-minimally coupled scalar fields (Bertolami & Martins 2000; Barrow & Parsons 1997), $f(R)$ theory (Tsujikawa 2008), conformal Weyl gravity (Mannheim 2000) and higher dimensional theories such as the Randall-Sundrum (RS) braneworld model (Randall & Sundrum 1999), and the braneworld model of Dvali-Gabadadze-Porrati (DGP) (Dvali et al. 2000).

Over the last few years considerable interest has been shown in the simple FRW linearly expanding (coasting) model in Einstein's theory with $a(t) \propto t$, $H(z) = H_0(1+z)$. Like the Λ CDM the total energy density and pressure in this model are expressed in terms of matter, radiation and dark energy components, such that $p = \omega\rho$ with $\rho = \rho_m + \rho_r + \rho_{de}$ and $p = p_r + p_{de}$ (since $p_m \approx 0$), but it includes the added assumption $\omega = -1/3$, i.e. the cosmic fluid acting as the source has zero active gravitational mass. So this would definitely exclude a cosmological constant as the source of the dark energy component in this case. The model was first discussed by Kolb (1989) who referred to this zero active mass cosmic fluid as “K-matter”. Interest in this model has been revived recently after it was noted (Melia 2003) that in the standard model the radius of the gravitational horizon $R_h(t_0)$ (also known as the Hubble radius) is equal to the distance ct_0 that light has traveled since the big bang, with t_0 being the current age of the universe. In the Λ CDM this equality is a peculiar coincidence because it just happens at the present time t_0 . It was then proposed (Melia 2007, 2009; Melia & Shevchuk 2012) that this equality may not be a coincidence at all, and should be satisfied at all cosmic time t . This was done by an application of Birkhoff's theorem and its corollary, which for a flat universe allows the identification of the Hubble radius R_h with the gravitational radius $R_h = 2GM/c^2$, given in terms of the Misner-Sharp mass $M = (4\pi/3)R_h^3(\rho/c^2)$ (Misner & Sharp 1964). The added assumption of a zero active gravitational mass $\rho + 3p = 0$ implies (Melia & Shevchuk 2012) that $R_h = ct$ or $H = 1/t$ for any cosmic time t . This linear model became known as the $R_h = ct$ universe. Unlike the Λ CDM/ ω CDM which contains at least the three parameters H_0 , Ω_m and ω_{de} , the $R_h = ct$ model depends only on the sole parameter H_0 , so that for example the luminosity distance used to fit Type Ia supernova data (Melia 2009) is given by the simple expression $d_L = (1+z)R_h(t_0)\ln(1+z)$. Also while the Λ CDM would need inflation to circumvent the well-known horizon problem, the $R_h = ct$ universe does not require inflation. One should also point out that the condition $R_h = ct$ is also satisfied by other linear models such as the Milne universe (Milne 1933), which however has been refuted by observations. Unlike the $R_h = ct$ model discussed here, the Milne universe is empty ($\rho = 0$) and with a negative spatial curvature ($k = -1$). As a result of these properties its luminosity distance is given by $d_L^{\text{Milne}} = R_h(t_0)(1+z)\sinh[\ln(1+z)]$, and it was shown that this is not consistent with observational data (Melia & Shevchuk 2012).

In the last few years the $R_h = ct$ universe received a lot of attention when it was shown (Melia & Maier 2013; Wei et al. 2013, 2014a,b, 2015; Melia et al. 2015) that it is actually favored over the standard Λ CDM (and its variant ω CDM with $\omega \neq -1$) by most observational data. This claim has

been contested by Bilicki & Seikel (2012) and Shafer (2015) who argued that measurement of $H(z)$ as a function of redshift and the analysis of Type Ia supernovae favored the Λ CDM over the $R_h = ct$ universe. However this was later contested by Melia & McClintock (2015) who showed that the $R_h = ct$ was still favored when using model-independent measurements that are not biased towards a specific model. Others (see for example van Oirschot et al. (2010); Lewis & van Oirschot (2012); Mitra (2014) have also criticized the model itself, particularly the validity of the EOS $\omega = -1/3$ (Lewis 2013). These and other criticisms have been addressed by Bikwa, Melia & Shevchuk (2012); Melia (2012) (see also Melia (2015) and references therein.) As pointed out above the $R_h = ct$ model would still require a dark energy component ρ_{de} , albeit not in the form of a cosmological constant. So the obvious question at this point would be: what are the possible sources for this component that together with the matter and radiation components will give the required total EOS, $\omega = -1/3$? The purpose of this paper is to answer this question by discussing the various possible sources of dark energy that are consistent with this EOS. Since the radiation component ρ_r at the present time t_0 is insignificant (at least for the Λ CDM with which this model has been compared) we assume that the total energy density $\rho = \rho_{de} + \rho_m$ and the total pressure $p = p_{de}$ ($p_m \approx 0$), as is normally done in the other alternative dynamical dark energy models found in the literature. So in the next three sections we examine three possibilities for the source of dark energy in the $R_h = ct$ model, namely a variable cosmological term $\Lambda(t)$, a non-minimally coupled scalar field in Brans-Dicke theory which is equivalent to a variable gravitational constant $G(t)$, and finally quintessence represented by a minimally coupled scalar field ϕ . We show that although the first two sources are consistent with the model, they are both unphysical, which leaves the third source of quintessence as the viable source of dark energy in the $R_h = ct$ universe. Results are then discussed in the Conclusion. Unless otherwise noted we use units such that $G = c = 1$.

2 Varying cosmological term $\Lambda(t)$

In this case Einstein's field equations take the form

$$R^{ij} - \frac{1}{2}Rg^{ij} = 8\pi \left[T^{ij} - \frac{\Lambda}{8\pi}g^{ij} \right], \quad (1)$$

where $\Lambda(t)$ is a time varying vacuum term representing the dark energy component of the source. In this case the time dependent vacuum satisfies the same EOS as in the Λ CDM, i.e., $w_{de} = p_{de}/\rho_{de} = -1$, but unlike the standard model the energy density $\rho_{de} = \Lambda(t)/8\pi$ is a function of the cosmic time. Taking the spatially flat FRW metric

$$ds^2 = -dt^2 + a(t)^2[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)], \quad (2)$$

where $a(t)$ is the scale factor, and the energy momentum tensor of a perfect fluid

$$T_{ij} = (\rho_m + p_m)u_i u_j + p_m g_{ij}, \quad (3)$$

where $u^i = [1, 0, 0, 0]$ is the four-velocity vector in comoving coordinates, the field equations in (1) give

$$\begin{aligned} \left(\frac{\dot{a}}{a}\right)^2 &= \frac{8\pi}{3}\rho_m + \frac{\Lambda}{3}, \\ \frac{\ddot{a}}{a} &= -\frac{4\pi}{3}(\rho_m + 3p_m) + \frac{\Lambda}{3}. \end{aligned} \quad (4)$$

These equations can also be combined into

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{2}{(1+3\omega_m)}\frac{\ddot{a}}{a} = \left(\frac{1+\omega_m}{1+3\omega_m}\right)\Lambda, \quad (5)$$

where $p_m = \omega_m \rho_m$. Using the Hubble parameter $H = \dot{a}/a$ this can also be written as

$$H^2 + \frac{2}{1+3\omega_m}(\dot{H} + H^2) = \left(\frac{1+\omega_m}{1+3\omega_m}\right)\Lambda. \quad (6)$$

From Bianchi identities we get

$$\dot{\rho}_m + 3(1+\omega_m)H\rho_m + \dot{p}_m = 0, \quad (7)$$

so that in the case of a time dependent cosmological term $\Lambda(t)$, the matter part cannot be conserved separately as in the Λ CDM where $\rho_m \sim a^{-3}$, and so there should be some energy exchange between the two components.

For the $R_h = ct$ model, $a(t) = t/t_0$ where t_0 is the current age of the universe, so that $H(t) = 1/t$. As usual for the matter part we can take $\omega_m = 0$, so that (6) gives a time decreasing vacuum term $\Lambda = 1/t^2$. The matter energy density is then obtained from (4) and is given by $\rho_m = \frac{1}{4\pi t^2}$, which shows that it is not separately conserved. In this case the cosmological parameters $\Omega_m = \frac{8\pi}{3H^2}\rho_m = 2/3$ and $\Omega_{de} = \Lambda/3H^2 = 1/3$ are constants. This contrasts with the current value of $\Omega_{m0} \approx 0.27$ which is used in the Λ CDM to adequately fit the data. Also a similar and more general cosmological model in which the cosmological term $\Lambda = 3\gamma H^2$ and $a(t) = [3(1-\gamma)H_0 t/2]^{2/3(1-\gamma)}$ was discussed by Basilakos, Plionis & Solà (2009) (see also Arcuri & Waga (1994)) and it was shown that for $\gamma \geq 1/3$, cosmic structures cannot be formed via gravitational instability in such models. This therefore suggests that a variable cosmological term (with $\Lambda = 1/t^2$ corresponding to $\gamma = 1/3$) may not be an appropriate explanation for dark energy in the $R_h = ct$ universe.

3 Varying gravitational term $G(t)$

An example of an alternative gravitational theory to general relativity, in which the gravitational constant G is a function of spacetime is given by Brans-Dicke (BD) theory (Brans & Dicke 1961). The original motivation for the introduction of BD-theory was to produce a theory that accommodates Mach's principle (Sciama 1953) which is not completely consistent with general relativity. The theory satisfies Dirac's Large Number Hypothesis (LNH) and the variable gravitational term $G \sim 1/\psi$ is expressed in terms of a scalar field. BD theory has seen a renewed interest due to its association with superstring theories, extra-dimensional theories and cosmological models with inflation or accelerated expansion (Callan et al. 1985; Duff, Khuri & Lu 1995; Banerjee & Pavon 2001; Fabris, Gonçalves & de Sá Ribeiro 2006). The action of BD-theory in the so called Jordan frame is given by

$$S^{(\text{BD})} = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[\psi R - \frac{\omega}{\psi} g^{cd} \nabla_c \psi \nabla_d \psi - V(\psi) \right] + S^{(\text{M})}, \quad (8)$$

where

$$S^{(\text{M})} = \int d^4x \sqrt{-g} \mathcal{L}^{\text{M}} \quad (9)$$

is the matter action and ω is the dimensionless Brans-Dicke parameter[†]. BD theory approaches general relativity in the limit $|\omega| \rightarrow \infty$ and $V(\phi) \rightarrow 0$. Moreover solar system constraints from the Cassini mission (Bertotti, Iess & Tortora 2003) imply that $\omega > 4 \times 10^4$. In our case we take the scalar field potential $V(\psi)$ to be zero. Varying the action with respect to the metric tensor g_{ab} gives

$$G_{ab} = \frac{8\pi}{\psi} T_{ab} + \frac{\omega}{\psi^2} \left(\nabla_a \psi \nabla_b \psi - \frac{1}{2} g_{ab} \nabla^c \psi \nabla_c \psi \right) + \frac{1}{\psi} (\nabla_a \nabla_b \psi - g_{ab} \square \psi), \quad (10)$$

where $\square \psi = g^{ab} \nabla_a \nabla_b \psi$ and

$$T_{ab} = \frac{-2}{\sqrt{-g}} \frac{\delta}{\delta g^{ab}} \left(\sqrt{-g} \mathcal{L}^{(\text{M})} \right). \quad (11)$$

Varying the action with respect to the scalar field gives

$$\frac{2\omega}{\psi} \square \psi + R - \frac{\omega}{\psi^2} \nabla^c \psi \nabla_c \psi = 0. \quad (12)$$

Taking the trace of (10), yields

$$R = \frac{-8\pi T^{(\text{M})}}{\psi} + \frac{\omega}{\psi^2} \nabla^c \psi \nabla_c \psi + \frac{3\square \psi}{\psi}, \quad (13)$$

such that the scalar field equation in (12) becomes

$$\square \psi = \frac{8\pi}{2\omega + 3} T^{(\text{M})}, \quad (14)$$

where $T^{(\text{M})} = T^\mu_\mu$ is the trace of the matter energy momentum tensor. Multiplying (10) by ψ and taking the covariant derivative leads to

$$8\pi T^a_{b;a} = -\frac{1}{2} \left(R - \frac{\omega}{\psi^2} \psi_{,a} \psi^{,a} + \frac{2\omega}{\psi} \square \psi \right) \psi_{,b}, \quad (15)$$

such that unlike the previous case (12) implies the separate conservation of the matter energy momentum tensor $T^a_{b;a} = 0$.

For the FRW metric in (2) and the matter energy momentum tensor in (3) the field equations in (10) and (14) lead to the following equations

$$\frac{3\dot{a}^2}{a^2} = \frac{8\pi}{\psi} \rho_m + \frac{\omega}{2} \frac{\dot{\psi}^2}{\psi^2} - \frac{3\dot{a}}{a} \frac{\dot{\psi}}{\psi}, \quad (16)$$

$$-\frac{2\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} = \frac{8\pi}{\psi} p_m + \frac{\omega}{2} \frac{\dot{\psi}^2}{\psi^2} + \frac{\ddot{\psi}}{\psi} + \frac{2\dot{a}}{a} \frac{\dot{\psi}}{\psi}, \quad (17)$$

$$\frac{\ddot{\psi}}{\psi} + 3\frac{\dot{a}}{a} \frac{\dot{\psi}}{\psi} = \frac{8\pi}{\psi} \frac{\rho_m - 3p_m}{3 + 2\omega}, \quad (18)$$

and the conservation of the matter energy momentum tensor gives

$$\dot{\rho}_m + (\rho_m + p_m) \frac{3\dot{a}}{a} = 0, \quad (19)$$

such that $\rho_m = \rho_{m0} a^{-3}$. Taking the case $p_m = 0$ and substituting (18) in (17) and using (16), we get

[†] In this section the Brans-Dicke parameter ω should not be confused with the EOS parameter ω for the $R_h = ct$ universe.

$$\frac{H\dot{\psi}}{\psi} - \frac{3\Omega_m H^2}{\psi} \left(\frac{1+\omega}{3+2\omega} \right) + \dot{H} + 3H^2 = 0, \quad (20)$$

where $\Omega_m = 8\pi\rho_m/3H^2$. For the $R_h = ct$ universe, with $a(t) = t/t_0$ and $\Omega_m = \Omega_{m0}a^{-1} = \Omega_{m0}H_0^{-1}t^{-1}$, Eq. (20) yields

$$\psi(t) = \frac{3\Omega_{m0}(1+\omega)}{H_0 t(3+2\omega)}. \quad (21)$$

It can easily be checked that this expression for the scalar field satisfies the complete set of field equations in (16)–(19) provided that the Brans-Dicke parameter $\omega = -2$, which means that $\psi(t) = 3\Omega_{m0}/2H_0 t$, such that the variable gravitational term $G \sim 1/\psi \sim t$ increases with cosmic time t , contrary to Dirac's LNH in which $G \sim 1/t$. Hence this, together with fact that the required value of ω does not satisfy the solar system constraint, imply that this variable G model in BD theory does not offer an appropriate explanation of the dark energy component in the $R_h = ct$ universe. It should be noted that the $R_h = ct$ model $a(t) = t/t_0$, is also a solution of BD-theory with a variable cosmological term, which is represented by using a scalar potential of the type $V(\psi) = 2\psi\Lambda(\psi)$ in (8). In this case (see for example Pimentel & Diaz-Rivera (1999)) we get a similar behavior for $G \sim 1/t$, while the cosmological term $\Lambda \sim 1/t^2$.

4 Quintessence

In scalar-tensor theory, quintessence, which is represented by a light scalar field ϕ which can be minimally, non-minimally or conformally coupled to gravity and having negative pressure, has long been considered (see for example Faraoni (2000) or Harko, Lobo & Mak (2014) for recent results) as a possible explanation of cosmic acceleration. The action of scalar-tensor theory is given by

$$S = \int L \sqrt{-g} d^4x, \quad (22)$$

with Lagrangian density

$$L = \left(\frac{1}{2\kappa} - \frac{\xi\phi^2}{2} \right) R - \frac{1}{2} g^{ab} \nabla_a \phi \nabla_b \phi - V(\phi) + L_M, \quad (23)$$

where $\kappa = 8\pi G = 8\pi$, ξ is the coupling constant between the scalar field and the geometry (with $\xi = 0$ representing minimal coupling), $V(\phi)$ is the scalar field potential, and L_M is the Lagrangian density associated for the matter distribution. The variation of the action in (22) with respect to the scalar field ϕ leads to the Klein-Gordon equation

$$\square\phi - \xi R\phi - \frac{dV}{d\phi} = 0, \quad (24)$$

while the variation of the action with respect to the metric g_{ab} gives

$$(1 - \phi^2 \xi \kappa)(R_{ab} - \frac{1}{2} g_{ab} R) = \kappa(\theta_{ab} + T_{ab}), \quad (25)$$

where

$$\theta_{ab} = \nabla_a \phi \nabla_b \phi - \frac{1}{2} g_{ab} \nabla^c \phi \nabla_c \phi - V g_{ab} + \xi(g_{ab} \square\phi^2 - \nabla_a \nabla_b \phi^2), \quad (26)$$

represents the energy momentum tensor corresponding to the scalar field ϕ and

$$T_{ab} = (\rho_m + p_m)u_a u_b + p_m g_{ab}. \quad (27)$$

Writing (25) in the form $G_b^a = \kappa(\tilde{\theta}_b^a + T_b^a)$, where $\tilde{\theta}_b^a = \theta_b^a + \xi\phi^2 G_b^a$ and using the contracted Bianchi identities $\nabla_a G_b^a = 0$, we get

$$\partial_b \phi (\square\phi - \xi R\phi - \frac{dV}{d\phi}) + \nabla_a T_b^a = 0, \quad (28)$$

so that by the Klein-Gordon equation in (24) we get the separate conservation of the matter energy momentum tensor T_{ab} .

For the FRW metric in (2) the field equations (24) and (25) give for the minimally coupled case ($\xi = 0$),

$$H^2 = \frac{8\pi}{3}(\rho_m + \rho_\phi), \quad (29)$$

$$\dot{H} + H^2 = -\frac{4\pi}{3}(\rho_m + \rho_\phi + 3p_\phi), \quad (30)$$

and

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \quad (31)$$

respectively, where for a dust matter distribution $p_m = 0$, $\rho_m = \rho_{m0}a^{-3}$ (which follows from the conservation of the matter energy momentum tensor) and $\rho_\phi = \dot{\phi}^2/2 + V(\phi)$ and $p_\phi = \dot{\phi}^2/2 - V(\phi)$. So for the $R_h = ct$ universe with $a(t) = t/t_0$, $H(t) = 1/t$ and $\Omega_m = \Omega_{m0}a^{-1}$, these equations can be easily solved to give,

$$\phi(t) = \frac{1}{\sqrt{\pi}} \ln[2\sqrt{H_0 t} + \sqrt{2}\sqrt{2H_0 t - 3\Omega_{m0}}] - \frac{\sqrt{2H_0 t - 3\Omega_{m0}}}{\sqrt{2\pi H_0 t}}, \quad (32)$$

and

$$V(t) = \frac{4H_0 t - 3\Omega_{m0}}{16\pi H_0 t^3}, \quad (33)$$

such that

$$\rho_\phi = \frac{3(H_0 t - \Omega_{m0})}{8\pi H_0 t^3}, \quad p_\phi = -\frac{1}{8\pi t^2}. \quad (34)$$

The scalar field is well defined and satisfies the weak energy condition $\rho_\phi \geq 0$ $p_\phi + \rho_\phi \geq 0$ provided that $t \geq 3\Omega_{m0}/2H_0$. The EOS of the dark energy component represented by the scalar field, is given by

$$\omega_\phi = p_\phi/\rho_\phi = \frac{-H_0 t}{3H_0 t - 3\Omega_{m0}}, \quad (35)$$

For large t when the scalar field dominates over the matter distribution $\omega_\phi \rightarrow -1/3$ in accordance with the EOS $p = -\rho/3$ between the total pressure and total energy density in the $R_h = ct$ universe.

5 Conclusion

In this paper we have examined possible sources for the dark energy component of the $R_h = ct$ universe, that are consistent with the total EOS $p = -\rho/3$. We have limited our study to second order theories. It should be pointed out that linear cosmology may also be a solution in higher order theories. For example a vacuum closed FRW model with $a(t) = t/\sqrt{3}$ in the absence of dust is a solution (Federbush 2000) of the gravitational theory with a curvature quadratic action density of the type $(-g)^{1/2}(R_{ik}R^{ik} + bR^2)$, where $b \neq -1/3$.

In our case for simplicity, and also due to the fact that at the present time the radiation component is insignificant,

we have assumed that the total pressure p and total energy density ρ are made up of matter and dark energy components. This assumption is also made in most dynamical dark energy models found in the literature that are based on one or more of the sources presented here. In our case we have found that a time dependent cosmological term $\Lambda(t)$ in general relativity or variable gravitational constant $G(t)$ in Brans-Dicke theory are not appropriate sources for the dark energy component in the $R_h = ct$ universe. As pointed out by Basilakos, Plionis & Solà (2009) the first model has problems with structure formation via gravitational instabilities, while the second model will require $G(t)$ to increase linearly with cosmic time and also requires a Brans-Dicke parameter $\omega = -2$, which is nowhere near the solar system constraint on ω . This will therefore leave quintessence in general relativity as a viable source of the dark energy component in the $R_h = ct$ universe, particularly for $t > 3\Omega_{m0}/2H_0$, where the minimally coupled scalar field (quintessence) is defined and satisfies the weak energy condition. As expected the variable EOS $\omega_\phi = p_\phi/\rho_\phi$ approaches the total EOS $\omega = p/\rho = -1/3$ of the $R_h = ct$ universe for late cosmic time when the scalar field dominates over the matter component. Moreover in this approach the matter and dark energy components are separately conserved as shown in the previous section. This would therefore negate the recent claim made by Lewis (2013) that the EOS $R_h = ct$ universe is inconsistent with $p/\rho = -1/3$, or the later claim made by Mitra (2014) who suggested that the metric in (2) with $a(t) = t/t_0$ is a static vacuum solution. Both claims have already been rebutted by Melia (2015) himself, but in this paper we go a step further and obtain a possible explanation of the dark energy component which fits well with the required EOS for the $R_h = ct$ universe.

REFERENCES

- Ade P.A.R. et al., 2014, *A&A*, 571, 23
 Arcuri R. C., Waga I., 1994, *Phys. Rev. D*, 50, 2928
 Armendariz-Picon C. et al., 2001, *Phys. Rev. D*, 63, 103510
 Banerjee N., Pavon D., 2001, *Class. Quantum Grav.*, 18, 593
 Barrow J.D., Parsons P., 1997, *Phys. Rev. D*, 55, 1906
 Basilakos S., Plionis M., Solà J., 2009, *Phys. Rev. D*, 80, 083511
 Basilakos S., 2015, *Mod. Phys. Lett. A*, 30, 1540031
 Bertolami O., Martins, P.J., 2000, *Phys. Rev. D*, 61, 064007
 Bertotti B., Iess L., Tortora P., 2003, *Nature*, 425, 374
 Bikwa O., Melia F., Shevchuk A. S. H., 2012, *MNRAS*, 421, 3356
 Bilicki M., Seikel M., 2012, *MNRAS*, 425, 1664
 Brans C., Dicke R. H., 1961, *Phys. Rev.*, 124, 925
 Callan C. G., Friedan D., Martinec E. J., Perry M. J., 1985, *Nucl. Phys. B*, 262, 593
 Duff M. J., Khuri R. R., Lu J. X., 1995, *Phys. Rep.*, 259, 213
 Dvali G.R., Gabadaze G., Poratti M., 2000, *Phys. Lett. B*, 484, 112
 Fabris J. C., Gonçalves S. V. B., de Sá Ribeiro R., 2006, *Grav. Cosmol.*, 12, 49
 Faraoni V., 1999, *Phys. Rev. D*, 62, 023504
 Federbush P., 2000, *arXiv:hep-th/0002137v1*
 Guth A.H., 1981, *Phys. Rev. D*, 23, 347
 Guth A.H., Kaiser D.I., Nomura Y., 2014, *Phys. Lett. B*, 733, 112
 Harko T., Lobo F.S.N., Mak M. K., 2014, *Eur. Phys. J. C*, 74, 2784
 Ibotombi Singh N., Bembem Devi Y., Surendra Singh S., 2013, *Astrophys. Space Sci.*, 345, 213
 Ijjas A., Steinhardt P.J., Loeb A., 2013, *Phys. Lett. B*, 723, 261
 Kazanas D., 1980, *ApJ*, 241, L59
 Knop R.A. et al., 2003, *ApJ*, 598, 102
 Kolb E.W., 1989, *ApJ*, 344, 543
 Lewis G. F., van Oirschot P., 2012, *MNRAS Lett.*, 423, 26
 Lewis G. F., 2013, *MNRAS*, 432, 2324
 Linde A., 1982, *Phys. Lett. B*, 108, 389
 Mannheim P.D., 2000, *Found. Phys.*, 30, 709
 Melia F., 2003, *The Edge of Infinity: Supermassive Black Holes in the Universe*. Cambridge Univ. Press, Cambridge, p. 119
 Melia F., 2007, *MNRAS*, 382, 1917
 Melia F., 2009, *Int. J. Modern Phys. D*, 18, 1113
 Melia F., 2012, *J. Cosmol. Astropart. Phys.*, 09, 029
 Melia F., Shevchuk A., 2012, *MNRAS*, 419, 2579
 Melia F., Maier R. S., 2013, *MNRAS*, 432, 2669
 Melia F., 2015, *MNRAS*, 446, 1191
 Melia F., Wei J.-J., Wu X., 2015, *AJ*, 149, 2
 Melia F., McClintock T.M., 2015, *AJ*, 150, 119
 Milne E.A., 1933, *Z. Astrophys.*, 6, 1
 Misner C.W., Sharp D.H., 1964, *Phys. Rev.*, 136, 571
 Mitra A., 2014, *MNRAS*, 442, 382
 Peebles P.J.E., Ratra B., 2003, *Rev. Mod. Phys.*, 75, 559
 Perlmutter S. et al., 1999, *ApJ*, 517, 565
 Pimentel L. O., Diaz-Rivera L. M., 1999, *Int. J. Mod. Phys. A*, 14, 1523
 Randall, L., Sundrum, R., 1999, *Phys. Rev. Lett.*, 83, 3370
 Ray S., Mukhopadhyay U., Dutta Choudhury S.B.D., 2007, *Int. J. Mod. Phys. D*, 16, 1791
 Ray S. et al., 2011, *Int. J. Theor. Phys.*, 50, 2687
 Riess A.G. et al., 1998, *AJ*, 116, 1009
 Riess A.G. et al., 2004, *ApJ*, 607, 665
 Sciama D. W., 1953, *MNRAS*, 113, 34
 Shafer D. L., 2015, *Phys. Rev. D*, 91, 103516
 Singh P., Sami M., Dadhich N., 2003, *Phys. Rev. D*, 68, 023522
 Tsujikawa S., 2008, *Phys. Rev. D*, 77, 023507
 van Oirschot P., Kwan J., Lewis G. F., 2010, *MNRAS*, 404, 1633
 Wei J.-J., Wu X., Melia F., 2013, *ApJ*, 772, 43
 Wei J.-J., Wu X., Melia F., 2014a, *MNRAS*, 439, 3329
 Wei J.-J., Wu X., Melia F., 2014b, *ApJ*, 788, 190
 Wei J.-J., Wu X., Melia F., 2015, *MNRAS*, 447, 479
 Weinberg S., 1989, *Rev. Mod. Phys.*, 61, 1

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